LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2015**

MT 6604/MT 5500- MECHANICS - II

Date : 15/04/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

Section – A

Answer ALL questions $(10\times 2 = 20)$

1. State the conditions for non-existence of centre of gravity.
2. Mention the differences between centre of gravity and centre of mass.
3. State the principle of virtual work.
4. Mention any two forces which can be ignored in forming the equation of virtual work.
5. Write down the components of radial and transverse directions.
6. Write down the scalar form of the equation of simple harmonic motion.
7. Define central orbit.
8. Write down the p-r equation of a hyperbola.
9. State the theorems of perpendicular axes.
10. Define kinetic energy.

Section – B

Answer any FIVE questions $(5\times 8 = 40)$

11. Find the centre of gravity of a solid hemisphere.
12. A solid hemisphere is supported by a string fixed to a point A on its rim and to a point O on a smooth vertical wall with which a curved surface of the sphere is in contact at P. If θ and φ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\varphi = \frac{3}{8} + \tan\theta.$
13. A uniform chain, of length 2l, is to be suspended from two points A and B in the same horizontal line so that either terminal tension is n times that at the lowest point. Show that the span AB must be $\frac{2l}{\sqrt{n^2-1}}\log_e\left(n+\sqrt{n^2-1}\right).$
14. Derive the equation of the common catenary in the form $y = c\cosh(x/c).$
15. Show that the resultant of the simple harmonic motions of the same period in the same straight line is also a simple harmonic motion. Find the amplitude and epoch.
16. Derive the differential of a central orbit in polar coordinates.

17. A particle describes a circular orbit under an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of the distance.

18. Find the moment of inertia of the right solid cone of height h and semi vertical angle α about its axis.

Section – C

Answer any TWO questions $(2\times 20 = 40)$

19.

a) A square hole is punched out of a circular lamina of diameter 'a' having a radius as its diagonal . Show that the centre of gravity of the remaining is at a distance $\frac{a}{8\pi - 4}$ from the centre of the circle.

b) Four rods, each of length a and weight w are smoothly joined together to form a rhombus ABCD, which is kept in shape by a light rod BD. The angle BAD is 60° and the rhombus is suspended in a vertical plane from A. Find the thrust in BD.

(10 + 10)

20.

a) Show that the length of an endless chain which will hang over a circular pulley of radius "a" so as to be in content with two-thirds of the circumference of the pulley is $a\left[\frac{3}{\log\left(2+\sqrt{3}\right)}+\frac{4\pi}{3}\right]$.

b) If a particle executing a S.H.M has velocities v_1 and v_2 when its distances from the mean position O are d_1 and d_2 respectively. Find the amplitude, period and the velocity when its distance from O is $\frac{1}{2}(d_1+d_2)$. (10 + 10)

21.

a) State and prove the parallel axes theorem on moment of inertia.

b) A solid sphere is rolling down a plane, inclined to the horizon at an angle α and rough enough to prevent any sliding. Find its acceleration. (10 + 10)

22.

a) Find the moment of inertia of a parabolic plate cut off by an ordinate at a distance h from the vertex, about the tangent at the vertex.

b) Find the moment of inertia of a hollow sphere about a diameter, its internal and external radii being b and a. (10 + 10)

$$$$$$$